

Mechanics of Fluids

Bernoulli's Equation & Its Applications

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BERNOULLI'S EQUATION AND ITS APPLICATIONS

TOTAL ENERGY OF A LIQUID PARTICLE IN MOTION

The total energy of a liquid particle, in motion, is the sum of its potential energy, kinetic energy and pressure energy. Mathematically,

Total energy

$$E = Z + \frac{v^2}{2g} + \frac{p}{\gamma} \quad [\text{N-m per unit weight of liquid}]$$

TOTAL HEAD OF A LIQUID PARTICLE IN MOTION

The total head of a liquid particle in motion, is the sum of its potential head, kinetic head and pressure head. Mathematically,

Total head

$$H = Z + \frac{v^2}{2g} + \frac{p}{\gamma} \quad [\text{m of liquid}]$$

Example

water is flowing through a pipe of 70 mm diameter under a gauge pressure of 3.5 bar, and with a mean velocity of 1.5 m/sec. Neglecting friction, determine the total head, if the pipe is 7 meters above the datum line.

Solution

Given that

Diameter of pipe = 70mm

Pressure, $p = 3.5 \text{ bar} = 3.5 \times 10^5 \text{ N/m}^2$

Velocity, $v = 1.5 \text{ m/sec}$

Datum head $Z = 7 \text{ meter}$

∴ Total head

$$H = Z + \frac{v^2}{2g} + \frac{p}{\gamma} = 7 + \frac{1.5^2}{2 \times 9.81} + \frac{3.5 \times 10^5}{9.81 \times 10^3} = 42.8 \text{ m}$$

It states "For a perfect incompressible liquid, flowing in a continuous stream, the total energy of a particle remains the same while the particle moves from one point to another". This statement is based on the assumption that there are no frictional losses in the pipe.

Mathematically

$$Z + \frac{v^2}{2g} + \frac{p}{\gamma} = \text{Constant}$$

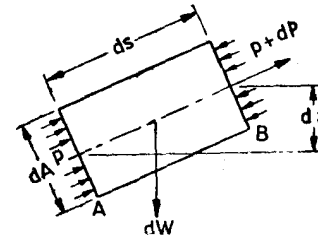
where Z = Potential energy per unit weight of flow, $\frac{V^2}{2g}$ = Kinetic energy per unit weight of flow and $\frac{P}{\gamma}$ = Pressure energy per unit weight of flow.

Euler's Equation for Motion

The Euler's equation for steady flow on an ideal fluid along a streamline based on the Newton's second law of motion. The integration of the equation gives Bernoulli's equation in the form of energy per unit weight of the fluid

flowing. It is based on the following assumptions:

- The fluid is non-viscous (i.e. the frictional losses are zero)
- The fluid is homogeneous and incompressible (i.e. $\rho = \text{constant}$)
- The flow is continuous, steady and along the streamline.
- The velocity of flow is uniform over the section.
- No energy or force, except gravity and pressure forces is involved in the flow.



Consider a steady flow of an ideal fluid along a streamline. Now consider a small element AB of the flowing fluid as shown in Figure.

Let dA = Cross - sectional area of the fluid element.

ds = Length of the fluid element.

dW = Weight of the fluid element.

p = Pressure on the element at A

$p + dp$ = Pressure on the element at B

v = Velocity of the fluid element

We know that the external forces tending to accelerate the fluid element in the direction of the streamline.

$$= p \cdot dA - (p + dp) \cdot dA = -dp \cdot dA \quad (i)$$

we also know that the weight of the fluid element

$$dW = \rho g \cdot dA \cdot ds$$

From the geometry of the figure, we find that the component of the weight of the fluid element in the direction of flow,

$$\begin{aligned} &= -\rho g \cdot dA \cdot ds \cos\theta \\ &= -\rho g \cdot dA \cdot ds \left(\frac{dz}{ds} \right) \qquad \left\{ \cos\theta = \frac{dz}{ds} \right\} \\ &= -\rho g \cdot dA \cdot dz \qquad \qquad \qquad \text{(ii)} \end{aligned}$$

$$\therefore \text{Mass of the fluid element} = \rho \cdot dA \cdot ds \qquad \qquad \qquad \text{(iii)}$$

We see that the acceleration of fluid element,

$$f = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v \cdot \frac{dv}{ds} \qquad \qquad \qquad \text{(iv)}$$

Now, as per Newton's second law of motion, we know that,

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$\text{or} \qquad -dp \cdot dA - \rho g \cdot dA \cdot dz = \rho \cdot dA \times v \frac{dv}{ds}$$

$$\text{or} \qquad \frac{dp}{\rho} + g \cdot dz = -v \, dv \qquad \qquad \qquad \{ \text{dividing both sides by } -\rho dA \}$$

$$\text{or} \qquad \frac{dp}{\rho} + g \cdot dz + v \, dv = 0 \qquad \qquad \qquad \text{(v)}$$

This is the required Euler's equation for motion, and is in the form of a differential equation.

Integrating the above equation,

$$\frac{1}{\rho} \int dp + g \cdot dz + \int v \cdot v \, dv = \text{constant}$$

$$\text{or} \qquad \frac{p^2}{\rho} + gZ + \frac{v^2}{2} = \text{constant}$$

$$\text{or} \qquad p + \gamma Z + \frac{\gamma v^2}{2g} = \text{constant}$$

dividing by ' γ '

$$\frac{p}{\gamma} + Z + \frac{v^2}{2g} = \text{constant}$$

or in other words

$$\frac{p_1}{\gamma} + Z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + Z_2 + \frac{v_2^2}{2g}$$

which proves *Bernoulli's* equation.

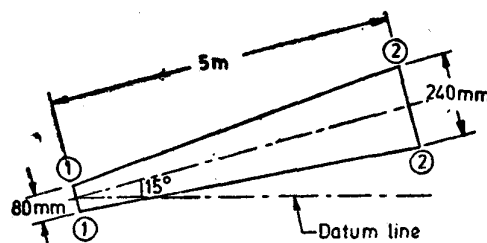
Limitations of Bernoulli's Equation

The Bernoulli's theorem has been derived on certain assumptions, which are rarely possible. Thus the Bernoulli's theorem has the following limitations;

- The Bernoulli's equation has been derived under the assumption that the velocity of every liquid particle, across any cross section of the pipe, is uniform. But in actual practice, it is *not* so. The velocity of liquid particle in the centre of a pipe is maximum, and gradually decreases towards the walls of the pipe duct to pipe friction. Thus, while using this equation, only the mean velocity of the liquid should be taken into account.
- The Bernoulli's equation has been derived under the assumption, that no external forces, except the gravity force is acting on the liquid. But in actual practice, it is not so. There are always some external forces (such as pipe frictions) acting on the liquid, which affect the flow of the liquid. Thus while using the Bernoulli's equation, all such external forces should be neglected. But if some energy supplied to, or extracted from the flow, the same should also be taken into account.
- The Bernoulli's equation has been derived, under the assumption, that there is no loss of energy of the liquid particle while flowing. But in actual practice, it is rarely so. In a turbulent flow some kinetic energy is converted into heat energy; and in a viscous flow some energy is lost due to shear forces. Thus, while using Bernoulli's equation all such losses should be neglected.
- If the liquid is flowing in a curved path, the energy due to centrifugal force should also be taken into account.

Example

A 5 meter long pipe is inclined at an angle of 15° with the horizontal. The smaller section of the pipe, which is at a lower level is of 80 mm diameter and the larger section of the pipe is of 240 mm diameter as shown in Figure. Determine the difference of pressures between the two section in N/m, if the pipe is uniformly tapering and the velocity of water at the smaller section is 1 m/sec.



From the equation of continuity, we know that

$$a_1 v_1 = a_2 v_2$$

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{\frac{\pi}{4} \times 0.08^2 \times 1}{\frac{\pi}{4} \times 0.24^2} = \frac{1}{9} \text{ m/sec}$$

Now, Applying Bernoulli's theorem for both the sections of the pipe,

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\gamma}$$

$$0 + \frac{1^2}{2 \times 9.81} + \frac{p_1}{9810} = (1.294 - 0) + \frac{\left(\frac{1}{9}\right)^2}{2 \times 9.81} + \frac{p_2}{9810}$$

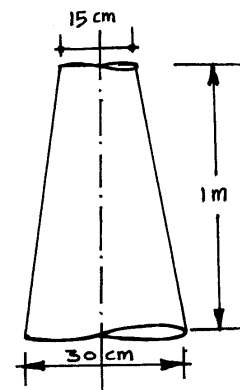
$$\Rightarrow \frac{(p_1 - p_2)}{9810} = 1.294 + \frac{1}{2 \times 9.81} \left(\left(\frac{1}{9}\right)^2 - (1)^2 \right)$$

$$\Rightarrow \frac{(p_1 - p_2)}{9810} = 1.244$$

$$\text{or } (p_1 - p_2) = 1.244 \times 9810 = 12200 \text{ N/m}^2 = 12.2 \text{ kN/m}^2$$

Example

Gasoline (Sp. gravity 0.8) is following upwards through a vertical pipe line which tapers from 300mm to 150mm diameter. A gasoline mercury differential manometer is connected between 300mm and 150mm pipe sections to measure the rate of flow. The distance between the manometer tapping is 1 meter and gauge reading is 500mm of mercury.



- (a) Find the gauge reading in terms of gasoline head.
- (b) using Bernoulli's equation and the equation of continuity, find the rate of flow. Neglect friction and other losses.

Solution

- (a) Differential gauge reading in terms of gasoline head

The gauge reading

$$= 500 \text{ mm of mercury} = \frac{13.6 - 0.8}{0.8} \times 500 \text{ mm of gasoline}$$

$$= 8000 \text{ mm} = 8 \text{ meter of gasoline}$$

From equation of continuity, we know that

$$a_1 v_1 = a_2 v_2$$

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{\frac{\pi}{4} \times (0.03)^2 \times v_1}{\frac{\pi}{4} \times (0.15)^2} = 4v_1$$

Now using Bernoulli's equation for inlet or outlet of the pipe

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\gamma}$$

or

$$Z_1 + \frac{v_1^2}{2g} + \left(\frac{p_1}{\gamma} - \frac{p_2}{\gamma} \right) = Z_2 + \frac{v_2^2}{2g}$$

$$0 + \frac{v_1^2}{2g} + 8 = 1 + \frac{(4v_1)^2}{2g} \quad \{ \text{Assuming } Z_1 = 0 \}$$

$$\frac{15v_1^2}{2g} = 7$$

$$\Rightarrow v_1 = \sqrt{\frac{7 \times 2 \times 9.81}{15}} = 3.026 \text{ m/sec}$$

$$\therefore \text{ Discharge, } Q = a_1 v_1 = \frac{\pi}{4} \times (0.03)^2 \times 3.026 = 0.214 \text{ m}^3/\text{sec}$$

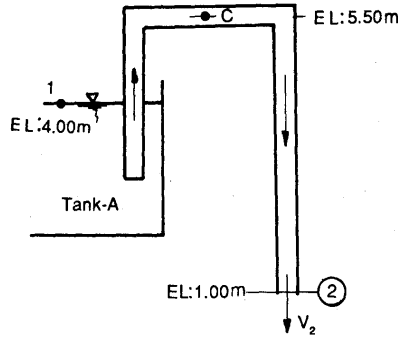
Example

A siphon consisting of a pipe of 15 cm diameter is used to empty kerosene oil ($RD = 0.80$) from a tank A. The siphon discharges to the atmosphere at an elevation of 1.00 m. The oil surface in the tank is at an elevation of 4.00 m. The centrelines of the siphon pipe at its highest point C is at an elevation of 5.50 m. Estimate (i) the discharge in the pipe, and (ii) pressure at point C. The losses in the pipe can be assumed to be 0.5 m up to the summit and 1.2 m from the summit to the outlet.

Solution

Consider points 1 and 2 at the surface of the oil in tank A and at the outlet as in fig. E-4.6. The velocity V_1 can be assumed to be zero. Between points 1 and 2, by Bernoulli equation we have

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_{L(1-2)}$$



$$0 + 0 + 4.00 = 0 + \frac{V_2^2}{2g} + 100 + (0.5 + 1.20)$$

$$\frac{V_2^2}{2g} = 4.00 - 2.70 = 1.30$$

$$V_2 = (2 \times 9.81 \times 1.30)^{1/2} = 5.05 \text{ m/s}$$

(i) Discharge = $Q = \frac{\pi}{4} \times (0.15)^2 \times 5.05 = 0.0892 \text{ m}^3/\text{s} = 89.2 \text{ L/s}$

(ii) Using suffix 3 to denote the conditions at the summit C, by applying Bernoulli equation to points 1 and 3,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + H_{L(1-3)}$$

$V_3 = V_2 = 5.05 \text{ m/s}$ as the pipe is of uniform cross section.

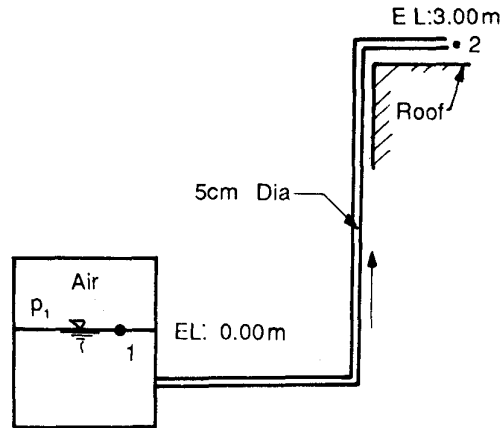
$$0 + 0 + 4.0 = \frac{p_3}{\gamma} + \frac{(5.05)^2}{2 \times 9.81} + 5.50 + 0.5$$

$$\frac{p_3}{\gamma} = 4.0 - 1.30 - 5.50 - 0.5 = -3.3 \text{ m}$$

$$p_3 = -33 \times (0.8 \times 998 \times 9.81) / 1000 = -25.85 \text{ KPa (gauge)}$$

Example

A closed tank containing water is partly filled with water and the air space it is under pressure. A 5 cm hose connected to the tank discharges water to atmosphere on to the roof of a building 3 m above the level of water in the tank. If friction losses are 1.5 m of water, what air pressure must be maintained in the tank to deliver 15 L/s to the roof?



Solution

Discharge $Q = 0.015 \text{ m}^3/\text{s} = AV_2 = \frac{\pi}{4}(0.05)^2 V_2$

$\therefore V_2 = 7.639 \text{ m/s}$

By applying Bernoulli equation to points 1 and 2

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

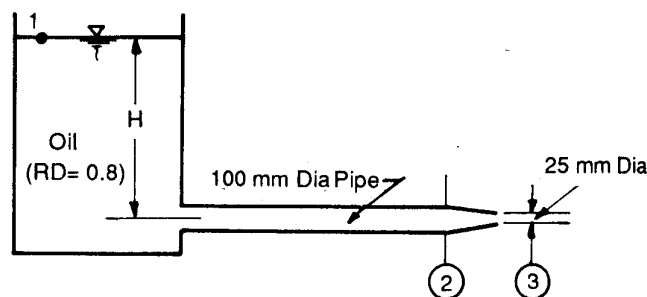
i.e. $\frac{p_1}{\gamma} + 0 + 0 = 0 + \frac{(7.639)^2}{2 \times 9.81} + 3.00 + 1.5$

Given $p_1/\gamma = 7.475$ and $\gamma = 9.79 \text{ kN/m}^3$

$\therefore p_1 = 9.79 \times 7.475 = 73.18 \text{ kPa}$

Example

Figure below shows a nozzle at the end of a pipe discharging oil from a tank to atmosphere. Estimate the discharge from the nozzle when the head H in the tank is 4.0 m. The loss in the pipe can be taken as $20 V^2/2g$ where V = velocity in the pipe. The loss of energy in the nozzle can be assumed to be zero. Also determine the pressure at the base of the nozzle.



By continuity criterion,

$$V_2 \frac{\pi}{4}(D)^2 = V_3 \frac{\pi}{4}(d)^2 = Q$$

Velocity in the pipe

$$V_2 = V_3 \left(\frac{d}{D} \right)^2 = \left(\frac{25}{100} \right)^2 V_3 = \frac{1}{16} V_3$$

Apply Bernoulli equation to points 1 and 3 with the centre line of the pipe as datum and atmospheric pressure as zero. The velocity at point 1 can be taken as zero.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + H_L$$

$$0 + 0 + H = 0 + \frac{V_3^2}{2g} + 0 + 20 \frac{V_2^2}{2g}$$

$$H = \frac{V_3^2}{2g} + 20 \times \left(\frac{1}{16} \right) h 2 \frac{V_3^2}{2g} = 1.07813 \frac{V_3^2}{2g}$$

As $H = 4.0 \text{ m}, V_3 = \left(\frac{2 \times 9.81 \times 4.0}{1.07813} \right)^{1/2} = 8.5319 \text{ m/s}$

Discharge $Q = \frac{\pi}{4} \times (0.025)^2 \times 8.5319 = 4.1881 \times 10^{-3} \text{ m}^3/\text{s} = 4.1881 \text{ L/s}$

$$V_2 = \text{Velocity in the pipe} = \frac{1}{16} \times 8.53188 = 0.5332 \text{ m/s}$$

Loss of head in the pipe $H_L = 20 \times \frac{V_2^2}{2g} = 20 \times \frac{(0.5332)^2}{2 \times 9.81} = 0.2899 \text{ m}$

Applying Bernoulli equation to points 1 and 2.

$$0 + 0 + H = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + 0 + H_L$$

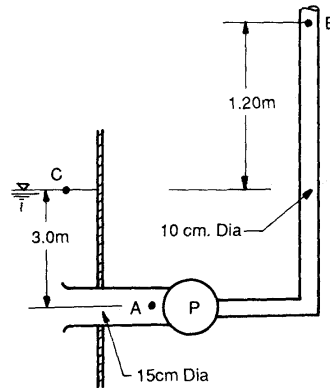
$$4.0 = \frac{p_2}{\gamma} + \frac{(0.5332)^2}{2 \times 9.81} + 0 + 0.2899$$

$$\frac{p_2}{\gamma} = 4.0 - 0.3044 = 3.6956 \text{ m}$$

$$\gamma = (0.8 \times 998 \times 9.81/1000) = 7.832 \text{ kN/m}^3$$

$$p_2 = \text{pressure at the base of the nozzle} \\ = 3.6956 \times 7.932 = 28.944 \text{ kPa}$$

Following figure shows a pump P pumping 60 L/s of water from a tank. (i) What will be the pressures at points A and B when the pump delivers 10 kW of power to the flow. Assume losses in the system to be negligible (ii) What will be the pressure at B when the loss in the inlet up to the pump is negligible and between the pump and the point B, a loss equal to 2 times the velocity head at B takes place.



Solution

Discharge $Q = 0.060 \text{ m}^3/\text{s} = A_a V_a = A_b V_b$

$$V_a = \frac{0.06}{\frac{\pi}{4} \times (0.15)^2} = 3.395 \text{ m/s}, \quad \frac{V_a^2}{2g} = 0.588 \text{ m}$$

$$V_b = \frac{0.06}{\frac{\pi}{4} \times (0.10)^2} = 7.639 \text{ m/s}, \quad \frac{V_b^2}{2g} = 2.975 \text{ m}$$

Power delivered by pump $P = \gamma Q H_p = 10 \text{ kW}$

$\therefore 9.79 \times 0.060 \times H_p = 10$

$H_p = 17.02 \text{ m} = \text{head delivered by the pump.}$

(i) By applying Bernoulli equation to points C and A

$$0 + 0 + 3 = \frac{p_a}{\gamma} + \frac{V_a^2}{2g} + 0$$

$$\frac{p_a}{\gamma} = 3 - 0.588 = 2.412 \text{ m}$$

$$p_a = 9.79 \times 2.412 = 23.61 \text{ KPa}$$

By applying Bernoulli equation between C and B with level at A as datum:

$$\frac{p_c}{\gamma} + \frac{V_c^2}{2g} + Z_c + H_p = \frac{p_b}{\gamma} + \frac{V_b^2}{2g} + Z_b$$

$$0 + 0 + 3.0 + 17.02 = \frac{p_b}{\gamma} + 2.975 + (3.0 + 1.2)$$

$$\frac{p_b}{\gamma} = 12.845 \text{ m and } p_b = 9.79 \times 12.845 = 125.75 \text{ kPa}$$

(ii) When losses are considered: By applying Bernoulli equation between C and B with level at A as datum,

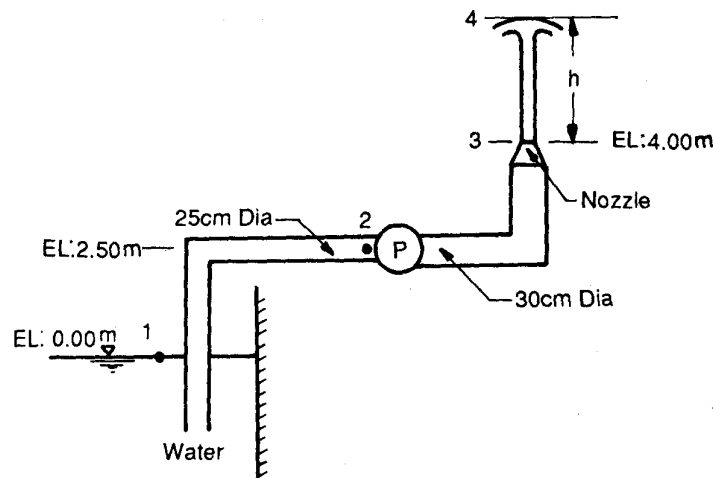
$$\frac{p_c}{\gamma} + \frac{V_c^2}{2g} + Z_c + H_p = \frac{p_b}{\gamma} + \frac{V_b^2}{2g} + Z_b + H_L$$

$$0 + 0 + 3.0 + 17.02 = \frac{p_b}{\gamma} + 2.975 + (3.0 + 1.2) + (3.0 + 1.2) + (2 \times 2.975)$$

$$\frac{p_b}{\gamma} = 6.985 \text{ m and } p_b = 9.79 \times 6.895 = 67.50 \text{ KPa}$$

Example

A pump is 2.5 m above the water level in the sump and has a pressure of -22 cm of mercury at the suction side. The suction pipe is of 25 cm diameter and the delivery pipe is a short 30 cm dia pipe ending in a nozzle of 10 cm diameter. If the nozzle is directed vertically upwards at an elevation of 4.0 m above the sump water level, determine (i) the discharge (ii) power input into the flow by the pump and the (iii) elevation above sump water level, to which the jet would reach. (Neglect all losses).



Solution

Applying Bernoulli's equation to points 1 and 2

$$0 + 0 + 0 = (-22 \times 13.6) + \frac{V_1^2}{2g} + 2.5$$

i.e. $V_1 = 3.107 \text{ m/s}$

$$Q = A_1 V_1 = (\pi/4)(0.25)^2 \times (3.107) = 0.1525 \text{ m}^3/\text{s} = 152.5 \text{ L/s}$$

$$V_3 = V_1 \times (D_1/D_3)^2 = 3.107 \times (25/10)^2 = 19.419 \text{ m/s}$$

$$\therefore V_3^2/2g = (19.419)^2/(2 \times 9.81) = 19.22 \text{ m}$$

Hence, the height to which the jet will reach $h = 19.22 \text{ m}$.

The elevation of point 4, the summit of the jet = $4 + 19.22 = 23.22 \text{ m}$

To find power delivered by the pump

$$0 + 0 + 0 + H_p = 0 + \frac{V_3^2}{2g} + 4$$

$$H_p = 19.22 + 4 = 23.22 \text{ m}$$

Power delivered by pump

$$P = 9.79 \times 0.1525 \times 23.22 = 34.67 \text{ kW}$$

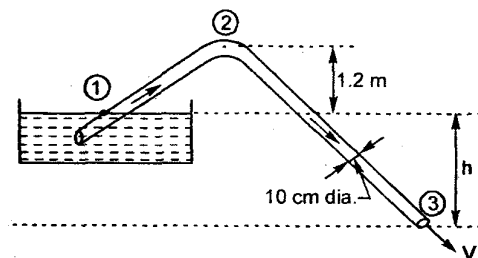
Example (AMIE Summer 2007, 10 marks)

A liquid of specific gravity 1.52 is discharged from a tank through a siphon whose summit point is 1.2 m above the liquid level in the tank. The siphon has a uniform diameter of 10 cm and it discharges the liquid into atmosphere whose pressure is 101 kPa. If the vapour pressure of the liquid is 28 kPa (abs), how far below the liquid level in the tank can the outlet be safely located? What is the maximum discharge? Neglect all losses of head.

Solution

Referring to figure, apply Bernoulli's equation on 2 and 3

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{p_3}{\rho g} + \frac{V_3^2}{2g} + Z_3$$



where $Z_3 = 0$, $V_2 = V_3 = V$, $p_3 = 101 \text{ kPa}$, $Z_2 = 1.2 + h$

$$\therefore \frac{28 \times 1000}{(1.52 \times 1000) \times 9.81} + \frac{V^2}{2g} + (1.2 + h) = \frac{101 \times 1000}{(1.52 \times 1000) \times 9.81} + \frac{V^2}{2g} + 0$$

Solving $h = 3.7$ m

This is the safe limit of h.

Applying Bernoulli's equation between points 1 and 3

$$\frac{101 \times 1000}{(1.52 \times 1000) \times 9.81} + 0 + 3.7 = \frac{101}{(1.52 \times 1000) \times 9.81} + \frac{V^2}{2g} + 0$$

Solving $V = 8.52$ m/s

$$\therefore Q = AV = \frac{\pi}{4} (0.1)^2 \times 8.52 = 0.0669 \text{ m}^3/\text{s}$$

Problem

A 15 cm diameter pipe is reduced to 7.5 cm diameter through a gradual contraction. At this contraction the difference between the piezometric heads at the main and the contracted section is 4 cm of mercury. By neglecting losses, calculate the discharge of water.

Answer: $Q = 14.9$ L/s

Problem

A conical tube is fixed vertically with its larger diameter at the top and forms a part of a pipeline carrying kerosene ($RD = 0.80$). The velocity at the smaller end is 3.0 m/s and at larger end is 1.5 m/s. The tube is 2 m long. At the bottom of the tube the pressure is 50 KPa. The head loss in the tube can be assumed to be 0.35 times the difference in the velocity heads at the two ends. Estimate the pressure at the top of the tube when the flow is upwards.

Answer: $p_2 = 36.09$ KPa

Problem

A pump draws from a sump whose water surface is 1.5 m below the centre of the pump and discharges it freely to atmosphere at 1.2 m above the pump centerline. The suction and delivery pipes are 20 cm and 25 cm in diameter respectively. If the pressure at the suction end of the pump is -2.0 cm of mercury (gauge) determine the discharge and power imparted by the pump. Neglect all losses.

Answer: $Q = 153.7$ L/s, $P = 4.81$ kW

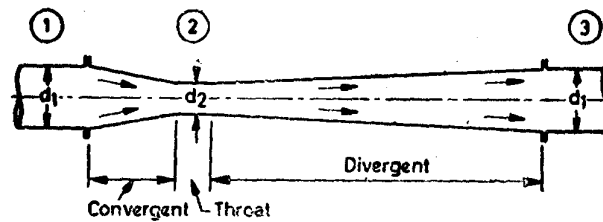
PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

The Bernoulli's equation is the basic equation which has the widest applications in Hydraulic and applied Hydraulics. Though it has a number of practical applications, yet we shall discuss its application on the following hydraulic devices :

1. Venturi meter
2. Orifice meter
3. Pitot tube

The venturi meter is an apparatus, for finding out the *discharge* of a liquid flowing in a pipe. A venturi meter in its simplest form, consists of the following three parts:

1. Convergent cone
2. Throat and
3. Divergent cone.



Convergent Cone

It is a short pipe which converges from a diameter d_1 (diameter of the pipe in which the venturi meter is fitted) to a smaller diameter d_2 (Diameter of throat).

Throat

It is a small portion of circular pipe of constant diameter d_2 .

Divergent Cone

It is a pipe, which diverges from a diameter d_2 to a larger diameter d_1 . Divergent cone is also known as outlet of the venturi meter. The liquid, while flowing through the convergent cone, is accelerated. As a result of the acceleration, the velocity of liquid at section 2 (i.e. at the throat) become higher than that at section 1. This increase in velocity, results in decreasing the pressure at section 2 considerably. If the pressure head drops at the throat below the separation head (which is 2.5 meters of water), then there will be a tendency of separation, of the liquid flow. In order to avoid this tendency there is always a fixed ratio of the throat diameter and the pipe (i.e., $\frac{d_2}{d_1}$). This ratio varies from 1/2 to 3/4, but the most suitable value

is 1/3 to 1/2.

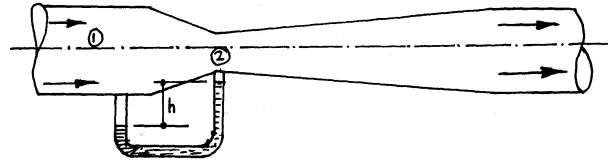
The liquid is decelerated (i.e. retarded) between the section 2 and 3 (i.e., while flowing through the divergent cone). As a result of this retardation, the velocity of liquid decreases which consequently increases the pressure. If the pressure is rapidly recovered, then there is every possibility for the stream of liquid to break away from the walls of the meter due to boundary layer effects. To avoid this tendency of breaking away, the divergent cone is made sufficiently longer. Another reason for making the divergent cone longer, is to minimize the frictional losses. Due to these reasons the divergent cone is 3 to 4 times longer than convergent cone as shown in figure.

Advantages are high pressure recovery, small loss of head, coeff. of discharge is high, not affected by wear and tear due to smooth surface, ideally suited for large flow rates.

Disadvantages are long length, more space required, expensive, not useful below 75 mm pipe diameter.

Discharge Through a Venturi Meter

consider a venturi meter, through which some liquid is flowing, as shown in Figure.



Applying Bernoulli's equation at section 1 and 2

i.e.
$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\gamma} \tag{i}$$

Let us pass our datum line through the axis of the venturi meter so that,

$$Z_1 = 0 \text{ and } Z_2 = 0$$

$$\therefore \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = \frac{v_2^2}{2g} + \frac{p_2}{\gamma}$$

or
$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \tag{ii}$$

Since the discharge at sections 1 and 2 is continuous, therefore

$$v_1 = \frac{a_2 v_2}{a_1}$$

$$\Rightarrow v_1^2 = \frac{a_2^2 v_2^2}{a_1^2} \tag{iii}$$

Substituting the above value of v_1^2 in equation (ii)

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{v_2^2}{2g} - \frac{a_2^2}{a_1^2} \times \frac{v_2^2}{2g} = \frac{v_2^2}{2g} \left(\frac{a_1^2 - a_2^2}{a_1^2} \right)$$

We know that $\frac{p_1}{\gamma} - \frac{p_2}{\gamma}$ is the difference between the pressure heads at sections 1 and 2. When

the pipe is horizontal, this difference represents the venturi head and is denoted by h,

or
$$h = \frac{v_2^2}{2g} \left(\frac{a_1^2 - a_2^2}{a_1^2} \right)$$

or
$$v_2^2 = 2gh \left(\frac{a_1^2}{a_1^2 - a_2^2} \right)$$

$$\Rightarrow v_2 = \sqrt{2gh} \left(\frac{a_1}{\sqrt{a_1^2 - a_2^2}} \right)$$

We know that the discharge through a venturi meter

$$Q = \text{Coefficient of venturi meter} \times a_2 v_2 = C a_2 v_2 = \frac{C a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

Note : The venturi head 'h' in the above equation has been taken in terms of the liquid head. But in actual practice, this head is given as the mercury head should be converted in to the liquid head by using the relation

$$h = \frac{13.6 - \gamma_{\text{liquid}}}{\gamma_{\text{liquid}}} \times \text{head of mercury}$$

where 13.6 is specific gravity of mercury, h = venturi head in terms of liquid head and γ_{liquid} = Specific weight of the liquid.

Example

A horizontal venturi meter 160 mm x 80 mm is used to measure the flow of an oil of sp. gravity 0.8. Determine the deflection of the oil-mercury gauge, if the discharge of the oil is 50 litres/sec.

Solution

Given

Diameter of pipe = 160mm = 16cm

∴ Area of pipe,

$$a_1 = \frac{\pi}{4} \times (16)^2$$

Diameter of throat = 80mm = 8cm

∴ Area of throat

$$a_2 = \frac{\pi}{4} \times 8^2 = 50.265 \text{ cm}^2$$

Sp. gravity of oil = 0.8

Discharge $Q = 50 \text{ liters/sec} = 50 \times 10^3 \text{ x cm}^3/\text{sec}$

Let h=Deflection of oil - mercury gauge in cm = $\left(\frac{13.6 \times 0.8}{0.8} \times h \right) = 16h \text{ cm of oil}$

and Coefficient of meter, C=1

Using the relation

$$Q = \frac{C_d a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$\Rightarrow 50 \times 10^3 = \frac{1 \times 201.06 \times 50.265}{\sqrt{(201.06)^2 - (50.265)^2}} \times \sqrt{2 \times 9.81 \times 16h}$$

$$= 9197 \sqrt{h} = 29.6 \text{ cm} = 296 \text{ mm}$$

Example

A venturimeter of throat diameter 5 cm is fitted into a 12.5 cm diameter water pipeline. The coefficient of discharge is 0.96. Calculate the flow in the pipeline when the reading on a mercury-water differential U-tube manometer connected to the upstream and throat sections shows a reading of 20 cm. If the energy loss in the downstream divergent cone of the meter is 10 times the velocity head in the pipe, calculate the total head loss of the meter.

Solution

(i) The venturimeter discharge is given by

$$Q = \frac{C_d A_2}{\sqrt{1 - (D_2/D_1)^4}} \sqrt{2g\Delta h}$$

The differential manometer reading y is related to Δh as

$$\Delta h = y \left(\frac{S_m}{S_p} - 1 \right) = 0.20 \left(\frac{13.6}{1.0} - 1 \right) = 2.52 \text{ m}$$

$$A_2 = \frac{\pi}{4} (0.05)^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\frac{D_2}{D_1} = \frac{5}{12.5} = 0.4$$

$$\sqrt{1 - \left(\frac{D_2}{D_1} \right)^4} = 0.9871$$

$$\therefore Q = \frac{0.96 \times 0.019635}{0.9871} \sqrt{2 \times 9.81 \times 2.52} = 0.01343 \text{ m}^3/\text{s} = 13.43 \text{ L/s}$$

$$(ii) V_1 = \text{Velocity in the pipe} = \frac{Q}{A_1} = \frac{0.01343}{(\pi/4)(0.125)^2} = 1.094 \text{ m/s}$$

$$\frac{V_1^2}{2g} = 0.061 \text{ m}$$

H_{Li} = head loss in the converging cone

$$= (1 - C_d^2) \Delta h = [1 - (0.96)^2] \times 2.52 = 0.1976 \text{ m of water}$$

H_{Ld} = head loss in the diverging cone

$$= 10(V_1)^2/2g = 10 \times 0.061 = 0.61 \text{ m of water}$$

Total head loss in the matter

$$H_L = H_{Li} + H_{Ld} = 0.1976 + 0.61 = 0.8076 \text{ m of water}$$

Problem

A vertical venturimeter 15 cm x 10 cm installed in a pipe carrying water downwards shows the same pressure at the inlet and at the throat. The throat is 25 cm below the inlet. If C_d of the meter is 0.95, calculate the discharge in the pipe.

Answer: $Q = 18.4 \text{ L/s}$

Problem

A horizontal venturimeter 9 cm x 4 cm is installed in a 9 cm water pipe. A differential water-mercury manometer reads 30 cm. If the coefficient C_d of the meter is 0.96, estimate the (i) discharge in the pipe and (ii) head loss in the converging section of the meter.

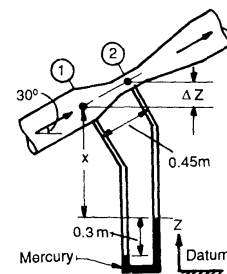
Answer: $Q = 10.6 \text{ L/s}$, $H_L = 0.296 \text{ m}$

Inclined venturimeter

Sometimes, a venturimeter is fitted to an inclined (or even a vertical pipe). The same formula for discharge, through the venturimeter (as we derived earlier) holds good. The discharge through an inclined venturimeter may also be found out, first by finding the velocity at either section (by using Bernoulli's equation) and then by multiplying the velocity with the respective area of flow.

Example

A pipe carrying water has a 30 cm x 15 cm venturimeter which is positioned inclined at 30° to the horizontal. The flow is upwards. The converging cone is 45 cm in length and the C_d of the meter is 0.98. A differential U-tube manometer with mercury as indicating fluid is connected to the inlet and to the throat and shows a differential column height of 30 cm (Fig. E - 13.21)



(i) Calculate the discharge in the pipe.

(ii) If the pressure in the inlet section is 50 kPa determine the pressure at the throat.

(iii) Find the head loss in the converging section of the venturimeter.

$$(i) \quad \left(\frac{p_1}{\gamma} + Z_1 \right) - \left(\frac{p_2}{\gamma} + Z_2 \right) = \Delta h$$

From differential manometer

$$\Delta h = y \left(\frac{S_m}{S_p} - 1 \right)$$

$$= 0.3 \left(\frac{13.6}{1} - 1 \right) = 3.78 \text{ m}$$

The discharge Q by the venturimeter equation is

$$Q = \frac{C_d A_2}{\sqrt{1 - (D_2/D_1)^4}} \times \sqrt{2g\Delta h}$$

$$C_d = 0.98,$$

$$A_2 = \frac{\pi}{4} \times \sqrt{(0.15)^2} = 0.01767 \text{ m}^2$$

$$\frac{D_2}{D_1} = \frac{15}{30} = 0.5 \sqrt{1 - (D_2/D_1)^4} = 0.96825$$

$$Q = \frac{0.98 \times 0.01767}{0.96825} \sqrt{19.62 \times 3.78} = 0.154 \text{ m}^3/\text{s} = 154 \text{ L/s}$$

$$(ii) \quad \Delta Z = Z_2 - Z_1 = \text{difference in elevation between the throat and the inlet}$$

$$= 0.45 \sin 30^\circ = 0.225 \text{ m}$$

$$\text{Since } \left(\frac{p_1}{\gamma} + Z_1 \right) - \left(\frac{p_2}{\gamma} + Z_2 \right) = \Delta h = 3.78 \text{ m}$$

$$\frac{p_2}{\gamma} = \frac{p_1}{\gamma} - (Z_2 - Z_1) - \Delta h = \frac{50}{9.79} - 0.225 - 3.78 = 1.102$$

$$p_2 = 1.102 \times 9.79 = 10.79 \text{ KPa}$$

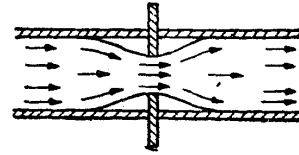
(iii) Head loss in the converging section

$$H_{Li} = \left(\frac{1}{C_d^2} - 1 \right) \left(1 - \left(\frac{D_2}{D_1} \right)^4 \right) \frac{V_2^2}{2g} = \left(\frac{1}{0.98^2} - 1 \right) \left\{ 1 - \left(\frac{15}{30} \right)^4 \right\} \left(\frac{0.154}{0.01767} \right)^2 \frac{1}{2 \times 9.81}$$

$$= 0.0387 \times 3.871 = 0.15 \text{ m}$$

$$\text{Alternatively: } H_L = (1 - C_d^2) \Delta h = (1 - 0.98^2) (3.78) = 0.15 \text{ m}$$

An orifice meter is used to measure the discharge in a pipe. An orifice meter, in its simplest form, consists of a plate having a sharp edged circular hole known as an orifice. This plate is fixed inside a pipe as shown in Figure.



A mercury manometer is inserted to know the difference of pressures between the pipe and the throat (i.e. orifice)

Let h = reading on the mercury manometer, p_1 is pressure at inlet, v_1 is velocity of liquid at inlet, a_1 is area of pipe at inlet and p_2, v_2, a_2 are corresponding values at throat.

Now applying Bernoulli's equation for inlet of pipe and throat

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\gamma} \quad (1)$$

or
$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad [Z_1 = Z_2]$$

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad (2)$$

Now we know from equation of continuity

$$a_1 v_1 = a_2 v_2$$

or
$$v_1 = (a_2 / a_1) v_2 \quad (3)$$

Putting this value of v_1 in (2)

$$h = \frac{v_2^2}{2g} \left(\frac{a_1^2 - a_2^2}{a_1^2} \right)$$

$\therefore v_2^2 = 2gh \cdot \left(\frac{a_1^2}{a_1^2 - a_2^2} \right)$

i.e.
$$v_2 = \sqrt{gh} \cdot \left(\frac{a_1}{\sqrt{a_1^2 - a_2^2}} \right)$$

Hence Discharge

$$Q = C \times a_2 v_2 = \frac{C a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh} \quad [C \text{ is coefficient of orifice meter}]$$

Length of orificemeter is short and hence it can be used in a wide variety of applications. Venturimeter has excessive length.

The disadvantage of orificemeter is that a sizeable pressure loss is incurred because of the flow separation down stream of the plate. In a venturimeter the gradual expanding section

keeps boundary layer separation to a minimum, resulting in good pressure recovery across the meter.

Advantages are low initial cost, ease in installation, less space, can be used in a wide range of sizes 910 mm to 1500 mm).

Example

An orifice meter consisting of 100 mm diameter orifice in a 250 mm diameter pipe has coefficient = 0.65. The pipe delivers oil of sp. gravity 0.8. The pressure difference on the two sides of the orifice plate is measured by a mercury oil differential manometer. If the differential gauge reads 800mm of mercury. Calculate the rate of flow in litre/sec.

Solution

Area of pipe section

$$a_1 = \frac{\pi}{4} \times (0.25)^2 = 0.049 \text{ m}^2$$

Area of orifice meter

$$a_2 = \frac{\pi}{4} \times (0.1)^2 = 0.0078 \text{ m}^2$$

differential gauge reading,

$$h = 800 \text{ mm of mercury} = 800 \times \left(\frac{13.6 - 0.8}{0.8} \right) = 12800 \text{ mm of oil} = 12.8 \text{ m of oil.}$$

Now using the relation

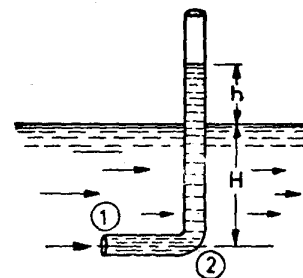
$$Q = \frac{C \cdot a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh} \text{ with usual notations}$$

$$Q = \frac{0.65 \times 0.049 \times 0.0078}{\sqrt{(0.049)^2 - (0.0078)^2}} \sqrt{2 \times 9.81 \times 12.8} = 0.08138 \text{ m}^3/\text{sec} = 81.38 \text{ liters/sec.}$$

PITOT TUBE

A Pitot tube is a simple instrument to determine the velocity of flow at the required point in a pipe or a stream. In its simplest form, a Pitot tube consists of a glass tube bent through 90° as shown in Figure.

The lower end of the tube faces the direction of flow as shown in the figure. the liquid rises up in the tube due to the pressure exerted by the flowing liquid. By measuring the rise of liquid in the tube, we can find out the velocity of the liquid flow.



Let, h = height of the liquid in the Pitot tube above the surface

H = depth of tube in the liquid, and

v = velocity of the liquid

Applying Bernoulli's equation for the sections 1 and 2

$$H + \frac{v^2}{2g} = H + h \quad (Z_1 = Z_2)$$

or
$$h = \frac{v^2}{2g} \Rightarrow v = \sqrt{2gh}$$

It has been experimentally found, that if the Pitot tube is placed with its nose facing side way, in the flow, there will be no rise of the liquid in the tube. But if a Pitot tube is placed, with its nose facing down stream, the liquid level in the tube will be depressed by an amount equal to 'h' such that,

$$h = \sqrt{\frac{v^2}{2g}} \quad \text{where 'v' is the velocity of the liquid flow.}$$

Example (AMIE S05, 10 marks)

A pitot tube is inserted in a pipe of 30 cm diameter. The static pressure in pipe is 10 cm of mercury (vacuum). The stagnation pressure at the centre of the pipe recorded by the pitot tube is 0.981 N/cm², calculate the rate of flow through pipe if the mean velocity of flow is 0.85 times the central velocity. Take $C_v = 0.98$

Solution

$$\frac{p_0}{\gamma} = \frac{p_{\text{static}}}{\gamma} = -\frac{10}{100} \times 13.6 = -1.36 \text{ m}$$

$$\frac{p_s}{\gamma} = \frac{p_{\text{stagnation}}}{\gamma} = \frac{1 \times 10^4}{9790} = 1.021$$

$$\Delta h = \left(\frac{p_s - p_0}{\gamma} \right) = [1.021 - (-1.36)] = 2.381 \text{ m}$$

$$v_m = \text{centreline velocity} = C \sqrt{2g\Delta h} = 0.98 \sqrt{2 \times 9.81 \times 2.381} = 6.693 \text{ m/s}$$

Mean velocity in the pipe = 0.85 x v_m

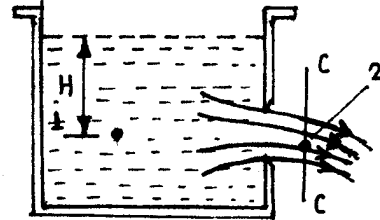
$$V = 0.85 \times 6.693 = 5.693 \text{ m/s}$$

and
$$Q = \frac{\pi}{4} \times (0.3)^2 \times (5.693) = 0.402 \text{ m}^3/\text{s}$$

An opening in a vessel, through which the liquid flows out known as an *orifice*. This hole or opening is called an orifice, so long as the level of the liquid on the upstream side is above the top of the orifice. The usual purpose of an orifice is the measurement of flow.

The continuous stream of a liquid, that flows out of an orifice, is known as the jet of water.

It has been observed that the jet after leaving the orifice, gets **contracted**. The maximum contraction takes place at a section slightly on the down stream side of the orifice, where the jet is more or less horizontal. Such a section is known as **vena contracta** as shown by section C-C in given figure. The distance of vena contracta depends upon the size of orifice and the head of water. Generally, this section is at a distance of about $d/2$ from the plane of orifice, where 'd' is the orifice diameter.



HYDRAULIC COEFFICIENTS

The following four coefficients are known as hydraulic coefficients or orifice coefficients.

1. Coefficient of velocity, C_v
2. Coefficient of contraction, C_c
3. Coefficient of discharge, C_d
4. Coefficient of resistance, C_r

Example

From Bernoulli's theorem derive the formula for the flow of a liquid through an orifice and introduce the coefficients of velocity, contraction and discharge.

Solution

Let us consider two points 1 and 2 as shown in fig. above. Consider flow to be steady and a constant head H.

Bernoulli's theorem

Applying Bernoulli's equation at points 1 and 2.

$$\frac{p_1}{w} + \frac{v_1^2}{2g} = \frac{p_2}{w} + \frac{v_2^2}{2g} \quad (\because z_1 = z_2)$$

Now $\frac{p_1}{w} = H$ and $\frac{p_2}{w} = 0$ (atmospheric pressure).

Since area of tank is very large as compared to the area of jet, hence v_1 is very small and can be easily neglected.

$$\therefore H + 0 = 0 + \frac{v_2^2}{2g}$$

or $v_2 = \sqrt{2gH}$; v_2 will give the theoretical velocity.

Coefficient of Velocity

The ratio of actual velocity of the jet, at vena contracta, to the theoretical velocity is known as coefficient of velocity. Mathematically,

$$C_v = \frac{\text{Actual velocity at vena contracta}}{\text{Theoretical Velocity}} = \frac{v}{\sqrt{2gH}}$$

where v = actual velocity and $\sqrt{2gH}$ = theoretical velocity

Value of C_v varies from 0.95 to 0.99.

Coefficient of Contraction

The ratio of area of the jet, at vena contracta, to the area of the orifice is known as coefficient of contraction. Mathematically,

$$C_c = \frac{\text{Area of jet at vena contracta}}{\text{Area of orifice}} = \frac{a_c}{a}$$

An average value of C_c is about 0.64.

Coefficient of Discharge

The ratio of actual discharge through an orifice to the theoretical discharge, is known as coefficient of discharge Mathematically,

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$
$$= C_v \times C_c$$

The value of coefficient of discharge varies with the values of C_c and C_v . An average is about 0.62.

Problem

A 60 mm diameter orifice is discharging water under a head of 9 metres. Calculate the actual discharge in litres/sec. and actual velocity of the jet in metres per second at vena contracta, if $C_d = 0.6$ and $C_v = 0.9$.

Solution

Let Q_{ac} = actual discharge through the orifice.

We know that the theoretical discharge

$$Q_{th} = a \sqrt{2gH} = 9\pi \sqrt{2 \times 981 \times 900} \text{ cm}^3/\text{sec.}$$

$$= 37570 \text{ cm}^3/\text{sec.} = 37.57 \text{ litres/sec.}$$

and $Q_{ac} = C_d \times C_{th} = 0.6 \times 37.57 = 22.54 \text{ litres/sec.}$

Actual velocity of the jet at vena contracta:

Let v_{ac} = actual velocity of jet at vena contracta.

We know that theoretical velocity of the jet

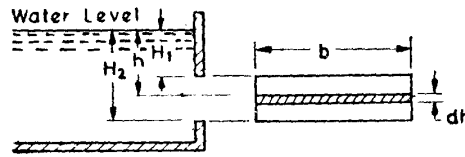
$$v_{th} = \sqrt{2gH} = \sqrt{2 \times 981 \times 900} = 1329 \text{ cm/sec.} = 13.29 \text{ m/sec.}$$

and $v_{ac} = C_v \times v_{th} = 0.9 \times 13.29 = 11.96 \text{ m/sec.}$

DISCHARGE THROUGH A LARGE RECTANGULAR ORIFICE

An orifice is considered to be larger, if the available head of the liquid is less than 5 times the height of the orifice.

Consider a large rectangular orifice, in one side of the tank, discharging water as shown in figure.



Let, H_1 = height of liquid above the top of orifice

H_2 = height of liquid above the bottom of the orifice

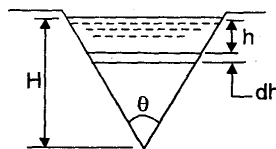
b = breadth of the orifice, and ; C_d = coefficient of discharge

Then, the total discharge through the whole orifice may be given by

$$Q = \frac{2}{3} C_d \cdot b \sqrt{2g} \left(H_2^{3/2} - H_1^{3/2} \right)$$

DISCHARGE THROUGH A TRIANGULAR NOTCH

Consider following figure.



Here H is head of water and θ is angle of notch.

Width of notch at any depth $h = 2(H - h)\tan\theta/2$. Consider an elemental horizontal strip of the opening at depth h and having a height dh . The theoretical velocity of flow through the strip = $\sqrt{2gh}$.

∴ Theoretical discharge through the strip

$$= 2(H-h) \tan \frac{\theta}{2} dh \sqrt{2gh}$$

Total discharge

$$Q = \int_0^H 2\sqrt{2g} \tan \frac{\theta}{2} (H-h) h^{1/2} dh = 2\sqrt{2g} \tan \frac{\theta}{2} (H-h) h^{1/2} dh$$

$$= 2\sqrt{2g} \tan \frac{\theta}{2} \left[H^{3/2} \frac{2}{3} H - \frac{2}{5} H^{5/2} \right] = \frac{8}{15} \sqrt{2g} \tan \frac{\theta}{2} H^{5/2}$$

Actual discharge

$$Q_{act} = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} H^{5/2}$$

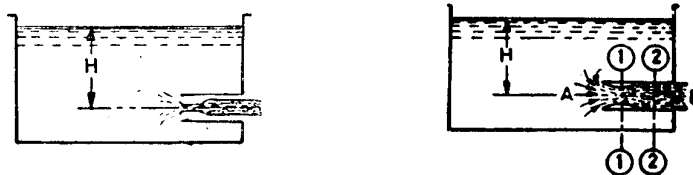
MOUTHPIECE

The discharge through an orifice depends upon its coefficient of discharge. Discharge through an orifice is too small due to low value of coefficient of discharge. If a short pipe is fitted to an orifice, it will increase value of coefficient of discharge, which will ultimately increase discharge. Such a pipe is called *mouthpiece* and its length is generally more than 2 times of orifice diameter. This pipe may be fitted externally or internally to the orifice.

An internal mouthpiece, extending into the fluid (i.e., inside the vessel), is known as Re-entrant or *Borda's* mouthpiece as shown in Figure (a) and (b). Following are the two types or internal mouthpieces. depending upon their nature of discharge:-

1. Mouthpiece running free, and
2. Mouthpiece running full.

If the jet, after contraction, does not touch the sides of the mouthpiece, it is said to be running free as shown in Figure (a). But if the jet, after contraction, expands and fills up the whole mouth-piece, it is said to be running full as shown in Figure (b)



(a) Mouthpiece running free (b) Mouthpiece running full

It has been experimentally found, that if the length of the mouthpiece is less than 3 times the diameter of the orifice, it will run free. But if the length of the mouthpiece is more than 3 times the diameter of the orifice, it will run full. The coefficient of discharge will be different, in both the cases.

MECHANICS OF FLUIDS
BERNOULLI'S EQUATION AND ITS APPLICATIONS
ASSIGNMENT

Q.1. (AMIE S13, 6 marks): From energy consideration, what is the important significance of potential head, datum head, and kinetic energy head. What is their relationship to total head ?

Q.2. (AMIE S06, 09, 10, 17, 10 marks): State and prove Bernoulli's equation. Mention its limitations.

Q.3. (AMIE S05, 10 marks): State Bernoulli's theorem for steady flow of an incompressible fluid. Derive expression for Bernoulli's equation from first principle and state the assumptions made for such a derivation.

Q.4. (AMIE W07, 15, 6 marks): Derive Bernoulli's equation from Euler's equation of motion.

Q.5. (AMIE W08, 09, 17, S12, 13, 16, 18, 10 marks): Derive Euler's equation of motion stating the assumptions. Obtain Bernoulli's equation from it.

Q.6. (AMIE S05, 10 marks): State the different devices that can be used to measure the discharge through a pipe also through an open channel. Describe one of such devices with a neat sketch and explain how one can obtain the actual discharge with its help.

Q.7. (AMIE S06, 10 marks): Describe how the pitot tube is used to determine the mass flow weighed mean value of the velocity in a large duct.

Q.8. (AMIE S13, 5 marks): Describe with the help of a neat sketch, the construction, operation and use of pitot static tube.

Q.9. (AMIE W09, 12, 5 marks): How will you determine the velocity of flow at any point with the help of pitot tube?

Q.10. (AMIE S06, 5 marks): What are the relative advantages and disadvantages of the orifice meter, venturimeter and the flow nozzle for measuring flow rates of gases?

Q.11. (AMIE W07, 10 marks): Write short notes on (i) Orifice meter (ii) Pitot tube

Q.12. (AMIE W08, 20 marks): Write short notes on (a) Venturimeter (b) Bernoulli's equation (c) Euler's equation (d) Pitot tube.

Q.13. (AMIE S15, 18, W15, 10 marks): What is a pitot tube? How is it used to measure velocity of flow at any point in a pipe or channel.

Q.14. (AMIE S13, W15, 5 marks): Compare and contrast the use of venturimeter, flow nozzle and orifice meter as primary element for flow measurement.

Q.15. (AMIE S08, 11, 16, 10 marks): Define an orificemeter. prove that the discharge through an orifice meter is given by

$$Q = C_d a_0 a_1 \sqrt{2gh} / \sqrt{a_1^2 - a_0^2}$$

where a_1 is area of pipe; a_0 is the area of orifice, and C_d is the coeff. of discharge.

Q.16. (AMIE S18, 10 marks): Explain the phenomenon of jet contraction in orifice flow.

Q.17. (AMIE W09, 12, S09, 12, 16, 10 marks): What is a venturimeter? Draw a neat sketch of the venturimeter showing the arrangement of the manometer. Derive an expression for the rate of flow of fluid through it.

Q.18. (AMIE S10, 8 marks): Define orifice, mouth piece, notch and weirs.

Q.19. (AMIE W05, 8 marks): Derive an expression for the discharge through a triangular notch. When would you recommend to use it.

Q.20. (AMIE S09, 6 marks): A horizontal water pipe of diameter 15 cm converges to 7.5 cm diameter. If the pressures at two sections are 400 kPa and 150 kPa, respectively, calculate the flow rate of water.

Answer: 0.102 m³/s

MECHANICS OF FLUIDS**BERNOULLI'S EQUATION AND ITS APPLICATIONS**

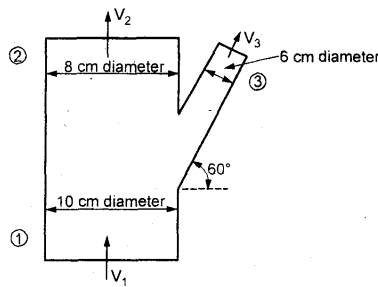
Q.21. (AMIE W06, 5 marks): Water flows in a circular pipe. At one section, the diameter is 0.2 m, the static pressure is 250 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. At a downstream section, the pipe diameter is 0.15 m and elevation is 0 m. Find the gauge pressure at the downstream section. Mention the assumptions.

Answer: 338.4 kPa, gauge

Q.22. (AMIE W08, 6 marks): Water flows through a 100 mm pipe at the rate of $0.027 \text{ m}^3/\text{s}$ and then through a nozzle attached to the end of the pipe. The nozzle tip is 50 mm in diameter, and the coefficients of velocity and contraction for the nozzle are 0.950 and 0.930, respectively. What pressure head must be maintained at the base of the nozzle if atmospheric pressure surrounds the jet?

Answer: 11.424 m

Q.23. (AMIE W06, 10 marks): Water flows through the horizontal Y branch shown in figure. For steady flow and neglecting losses, determine the force components required to hold the body Y in place.



The gauge pressure at section (1) is 30 kPa with volumetric inflow $15 \times 10^{-3} \text{ m}^3/\text{s}$, volumetric outflow from section (2) equals to $10^{-2} \text{ m}^3/\text{s}$ and water density is 1000 kg/m^3 .

Answer: $F_x = -47.2 \text{ N}$ (in $-x$ direction), $F_y = 1.172 \text{ N}$

Q.24. (AMIE S09, 10 marks): A submarine moves horizontally in sea with its axis much below the surface of water. A pitot tube, properly placed just in front of the submarine and along its axis, is connected to two limbs of a U tube containing mercury. The difference of mercury level is found to be 17 cm. Find the speed of the submarine knowing that density of mercury is 13.6 and that of sea water is 1.026 with respect to freshwater.

Answer: 22.556 km/h

Q.25. (AMIE W05, 8 marks): A pipeline carrying oil of specific gravity 0.87, changes in diameter from 200 mm at a position A to 500 mm at a position B, which is 4 m at a higher level. If the pressures at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively and the discharge is 200 litre/s, determine the loss of head and direction of flow.

Answer: A to B, 2.608 m

Q.26. (AMIE S13, 8 marks): The rate of water through a vertical conical draft tube of a Kaplan tube is $17.5 \text{ m}^3/\text{s}$. The diameter of the draft tube on the side connected to the outlet of the turbine runner is 2.5 m and the average velocity at exit is 1.5 m/s. If the pressure at inlet to the tube is not to be less than -0.7 bar, how far the tube should extend above the tail race. Neglect frictional effects and presume that exit of the draft tube lies 1.2 m below the tail water level.

Answer: 6.6 m

Q.27. (AMIE W12, 7 marks): A 30 cm diameter horizontal pipe terminates in a nozzle with the exit diameter of 7.5 cm. If the water flows through the pipe at a rate of $0.15 \text{ m}^3/\text{s}$, what force will be exerted by the fluid on the nozzle?

Answer: 35.80 kN

MECHANICS OF FLUIDS**BERNOULLI'S EQUATION AND ITS APPLICATIONS**

Q.28. (AMIE W07, 6 marks): A venturi meter, having a diameter of 75 mm at the throat and 150 mm diameter at the enlarged end, is installed in a horizontal pipeline 150 mm in diameter carrying an oil of specific gravity 0.9. The difference in pressure head between the enlarged end and the throat recorded by on U tube is 175 mm of mercury. Determine the discharge through the pipe. Assume the coefficient of discharge of the meter as 0.97.

Answer: 0.0308 m³/s

Q.29. (AMIE W07, 6 marks): A venturimeter of 30 cm inlet diameter and 15 cm throat diameter is provided in a vertical pipeline carrying oil of specific gravity 0.9, the flow being upward. The difference in elevation of the throat section and entrance section of the venturimeter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Calculate the (i) discharge of oil, (ii) pressure difference between the entrance section and the throat section. Take coefficient of meter as 0.98 and specific gravity of mercury as 13.6.

Answer: 148.795 l/s, 3.379 N/cm²

Q.30. (AMIE S10, 12 marks): A venturi meter is to be fitted in a pipe of 0.25 m diameter, where the pressure head is 7.6 m of following liquid and the maximum flow is 8.1 m³/min. Find the least diameter of the throat to ensure that the pressure head does not become negative. Take $C_d = 0.96$.

Answer: 11.94 cm

Q.31. (AMIE S13, 6 marks): A venturimeter with 200 mm at inlet and 100 mm throat is laid with axis horizontal, and is used for measuring the flow of oil of specific gravity 0.8. The difference of levels in the V-tube differential-manometer reads 180 mm of mercury whilst 11.52 x 10³ kg of oil is collected in 4 min. Calculate the discharge coefficient for the meter. Take specific gravity of mercury as 13.6.

Answer: 0.9845

Q.32. (AMIE W12, 5 marks): In a 100 mm diameter horizontal pipe, a venturi meter of 0.5 contraction ratio has been fitted. The head of water on the meter/when there is no flow, is 3 m (gauge). Find the rate of flow for which the throat pressure will be 2 m of water absolute. Take discharge coefficient for the meter as 0.97.

Answer: 28.4 l/s

Q.33. (AMIE S11, 10 marks): Find the discharge of water flowing through a pipe of 30 cm diameter placed in an inclined position where a venturimeter is inserted, having throat diameter of 15 cm. The difference of pressure between the main throat is measured by a liquid of specific gravity 0.6 in an inverted U tube which gives a reading of 30 cm. The loss of head between the main and throat is 0.2 times the kinetic head of the pipe.

Answer: 27.81 l/s

Q.34. (AMIE S08, 6 marks): An orifice meter with diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of specific gravity 0.9 when the coefficient of discharge of the meter is 0.64.

Answer: 137.42 l/s

Q.35. (AMIE S08, 12, 6 marks): An orifice meter, with orifice diameter 10 cm, is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter given readings of 19.62 N/cm² and 9.81 N/cm², respectively. Coefficient of discharge for the meter is given as 0.6. Find the discharge of water through pipe.

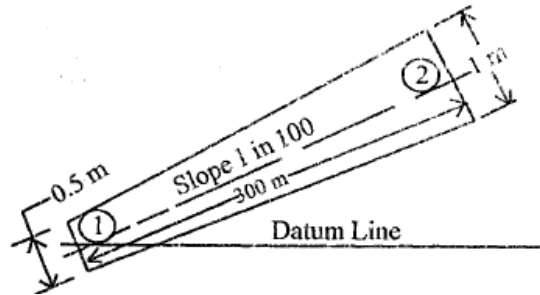
Answer: 68.21 l/s

Q.36. (AMIE W16, 10 marks): A 2 m long pipeline tapers uniformly from 10 cm diameter to 20 cm diameter at its upper end. The pipe is fitted inclined so that the axis of the pipe makes 30° angle with the horizontal line and water flows up from smaller end to bigger end. If the pressure gauge installed at the smaller end and upper

ends of the pipeline read 200 kPa and 230 kPa respectively, determine the flow rate and the field pressure at the mid length of the pipeline where the diameter is 15 cm.

Answer: $0.0724 \text{ m}^3/\text{s}$; 229 kPa

Q.37. (AMIE S17, 10 marks): A pipe 300 metres long has a slope of 1 in 100 and tapers from 1 metre diameter at the higher end to 0.5 metre at the lower end as shown in figure. The quantity of water flowing is 900 litres/second. If the pressure at the higher end is 70 kPa find the pressure at the lower end.



Answer: 89.58 kPa

Q.38. (AMIE S15, 10 marks): A venturi meter is installed in a pipeline carrying water and is 30 cm in diameter. The throat diameter is 12.5 cm. The pressure in pipeline is 140 kN/m^2 and the vacuum in the throat is 37.5 cm of Hg. Four percent of the differential head is lost between the gauges. Working from first principles, find the flow rate in the pipeline (in l/s), assuming the venturi meter to be horizontal.

Answer: $0.236 \text{ m}^3/\text{s}$

Q.39. (AMIE S16, 10 marks): A horizontal venturi meter 20 cm x 10 cm is used to measure the flow of oil of specific gravity 0.7. Determine the deflection of the oil mercury gauge, if the discharge of oil is 6017 s . Assume coefficient of discharge as 1.0. If the deflection of mercury gauge is 0.2 cm, find coefficient of the venturi meter.

Answer: 15.13 cm; 0.972

Q.40. (AMIE W16, 10 marks): Determine the rate of flow of water through a pipe 300 mm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 150 mm. The difference of pressure between the main and throat is measured by a liquid of specific gravity 0.7 in an inverted U-tube which gives a reading of 260 mm. The loss of head between the main and throat is 0.3 times the kinetic head of pipe.

Answer: $0.0232 \text{ m}^3/\text{s}$

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